SVKM's

D. J. Sanghvi College of Engineering

Program: B.Tech in Electronics & Telecommunication Engineering **Academic Year: 2022** **Duration: 3 hours**

Date: 19.01.2023

Time: 09:00 am to 12:00 pm

Subject: Engineering Mathematics III (Semester III) **Marks: 75**

Instructions: Candidates should read carefully the instructions printed on the question paper and on the cover page of the Answer Book, which is provided for their use.

1. This question paper contains two pages.
2. **All Questions are Compulsory.**
3. All questions carry equal marks.
4. **Answer to each new question is to be started on a fresh page.**
5. **Figures in the brackets on the right indicate full marks.**
6. **Assume suitable data wherever required, but justify it.**
7. Draw the neat labelled diagrams, wherever necessary.

Question No.		Max. Marks
Q1 (a)	Find the Laplace transform of $$\int_0^\infty e^{-t}\left(\int_0^t u^2 \sin hu \cos hu \ du\right) dt$$	[07]
	OR	
(a)	Evaluate $\int_0^\infty e^{-t}\frac{\sin^2 t}{t}\, dt$ by using Laplace transform.	[07]
Q1 (b)	i. Evaluate $L\{\frac{e^{-t}\sin 2t \ \sin h\, t}{t}\}$	[04]
	ii. Evaluate $L\{\sin 2t \ \cos t \ \cos h\, 2t\}$	[04]
Q2 (a)	Solve using Laplace Transform $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t}$ given $y(0) = 4$ *and* $y'(0) = 2$	[07]
	OR	
(a)	Find the inverse Laplace transform of $$\frac{s^2 + 16s - 24}{s^4 + 20s^2 + 64}$$	[07]
Q2 (b)	i. Find $L^{-1}\left\{\frac{s^2}{(s^2-a^2)^2}\right\}$ by using Convolution theorem.	[04]
	ii. Find inverse Laplace transform of	[04]

	$L^{-1}\left\{\tan^{-1}\frac{(s+a)}{b}\right\}$	
Q3 (a)	i. Find the Fourier series for $f(x)=\frac{3x^2-6x\pi+2\pi^2}{12}$ $in\ (0,2\pi)$. Hence deduce that $\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$.	[07]
	OR	
(a)	Find a half range cosine series of $f(x)=\sin x$ in $0\le x\le\pi$.	[07]
Q3 (b)	i. Obtain the complex form of Fourier series for $f(x)=e^{-ax}$ $in\ (-\pi,\pi)$.	[04]
	ii. Verify the set of functions $\{\sin x, \sin 3x, \sin 5x, \dots\}$ i.e. $\sin(2n+1)x$ $n+0,1,2,\dots$ is orthogonal or not over $\left[0,\frac{\pi}{2}\right]$. If orthogonal then construct the orthonormal set of functions.	[04]
Q4 (a)	Verify Green's theorem in the plane for $\int_C (xy+y^2)dx+x^2\,dy$ where C is the closed curve of the region bounded by $y=x$ and $y=x^2$.	[07]
	OR	
(a)	Use Stokes' theorem to evaluate $\int_C \overline{F}.d\overline{r}$ where $\overline{F}=(y^2+z^2-x^2)\mathrm{i}+(z^2+x^2-y^2)\mathrm{j}+(x^2+y^2-z^2)\kappa$ over the boundary of the surface $x^2+y^2-2ax+az=0$ above the plane $z=0$.	[07]
Q4 (b)	Find $div\,\overline{F}$ and $curl\,\overline{F}$ where $\overline{F}=\frac{xi-yj}{x^2+y^2}$.	[08]
Q5 (a)	By Milne-Thompson method find the imaginary part of the analytic function whose real part is $e^{2x}(x\cos 2y-y\sin 2y)$ and also verify that v is harmonic.	[07]
	OR	
(a)	Find the analytic function $f(z)=\frac{\cos x+\sin x-e^{-y}}{2\cos x-e^{y}-e^{-y}}$ $when$ $f\left(\frac{\pi}{2}\right)=0$.	[07]
Q5 (b)	i. Find the constants a, b, c, d, e if $f(z)=(ax^3+bxy^2+3x^2+cy^2+x)+i(dx^2y-2y^3+exy+y)$ is analytic.	[04]
	ii. Find the Bilinear Transformation which maps the points 2, i, -2 of z-plane onto the points 1, i, -1 of w-plane by using the cross-ratio property.	[04]